Multi-scale time step selection

Using the result from the 2D and 3D Douglas ADI,

Let

Denote the discretization error. Then

Assume the discretization error takes the form

Since the equation is linear (the behavior of each term of the series is the same as series itself), it is enough to consider the growth of a typical term:

With

Looking back to

Noting that

And

Plugging this in, and canceling the cosine factors, we have

The amplification factor, is then

Using this

Solving for yields

Or

Multiplying by we have